Abstracts

Sierpinski carpets, systems of low complexity and generalized continued fractions
Pierre ARNOUX

We study a family of simple dynamical systems of low complexity, depending on a parameter. The dynamics of these systems is best studied by using an induction operation, which leads to a generalized continued fraction algorithm. This continued fraction is defined on a subset of the standard simplex homeomorphic to the sierpinski carpet, which is known to be of Lebesgue measure 0, and which already appeared in various works; it would be interesting to know the Hausdorff dimension of this set, and to be able to compute the Gauss measure for the continued fraction which is absolutely continuous with respect to the Hausdoff measure.

Continuous Gaussian processes with random local Hölder regularity
Antoine AYACHE

Let $\{X(t)\}_{t\in\mathbb{R}}$ be an arbitrary Gaussian process whose trajectories are, with probability 1, continuous nowhere differentiable functions. It follows from a classical result, which is derived from zero-one law, that, with probability 1, the trajectories of X have the same global Hölder regularity over any compact interval i.e. the global Hölder regularity (uniform Hölder exponent) does not depend on the choice of the trajectory. A natural question which can be addressed is that whether or not this is also the case for the local Hölder regularity (pointwise Hölder exponent) of the trajectories of X. In this talk, using the framework of multifractional processes, we construct a family of counterexamples showing that the answer to this question is negative.

The local geometry of self-affine sets
Christoph BANDT

While the structure of self-similar sets with open set condition does not change under magnification, self-affine sets can undergo a metamorphosis when they are magnified.

In the first part of the talk we show that for certain self-affine Cantor sets in the plane, all tangent sets have a product structure with connected fibres: interval times Cantor set, like attractors of differentiable dynamical systems. This is work with Antti Käenmäki. In a second part we show that there are many smooth self-affine curves in the plane, but no C^2 curves, except for parabolic arcs. This was worked out with Aleksey Kravchenko. Finally, we shall discuss self-affine surfaces.

On the graphs of non-differentiable functions of the Weierstrass type
Krzysztof BARANSKI

We determine the Hausdorff and box dimension of the fractal graphs for a general class of Weierstrass-type functions of the form $f(x)=\sum_{n=1}^{\infty}a_n\,g(b_nx+\theta_n)$, where g is a periodic Lipschitz real function and $a_{n+1}/a_n\to 0$, $b_{n+1}/b_n\to\infty$ as $n\to\infty$. Moreover, for any $H,B\in[1,2]$, $H\le B$ we provide examples of such functions with $\dim_H(\mathrm{graph}(f))=\underline{\dim}_B(\mathrm{graph}(f))=H$, $\overline{\dim}_B(\mathrm{graph}(f))=B$.

Intermittent Random transport : the partially reflected brownian motion case

Athanasios BATAKIS

(Joint work with V.H. Nguyen)

We propose a simple model for intermittent random transport, related to problems arising from biology and physics. We are interested in the exit distribution of the relevant random processes.

Quantitative recurrence properties in beta dynamical system and continued fraction system

Véronique BILLAT [c].

(joint work with Eva THEUMANN-WESFREID [a,b])

The aim of this study was to detect randomness in heart rate (HR) and respiratory frequency (Rf) during a high altitude mountain ascent, by investigating the changes in the fractal scaling and time-frequency behavior and the entropy of HR and Rf with altitude. In order to analyze simultaneously these two signals, we first used the local cosine4 orthonormal bases whose spectrum is not redundant as those computed with the short Fourier transform. The signal energy is therefore split into local cosine4 spectrum energies over time segmented boxes.We performed then, the detrended fluctuations and the wavelet leader scaling analysis (DFA, WLA) to estimate HR and Rf scaling exponents. Results showed that HR and Rf were differently affected by altitude. Indeed, the average HR was not modified by altitude (p=0.18) in contrast to Rf which increased significantly (p = 0.0003). The variability of HR was altered during the mountain ascent after 3000m of altitude, the short range HR DFA (1.0 - 1.5) exponents increased after the altitude of 3000m (p¡0.03). In contrast, the Rf DFA short range exponents (0.5 – 1), were not affected by altitude (p=0.67). The ratio of low frequencies (0.94%) cosine4 spectrum energy over the local spectrum energy of HR did not change with altitude (p=0.11) in contrast with the Rf which decreased with altitude from 71 to 65

Keywords: Fractal analysis; Detrended Fluctuation Analysis (DFA); Wavelet; Heart rate, Entropy, Hypoxia, Fatigue.

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Multifractal spectrum of generic measures and functions monotone in several variables

Zoltan BUCZOLICH

(Joint work with Stéphane Seuret)

We prove that in the Baire category sense, measures supported by the unit cube of $\mathbb{R}^d$ typically satisfy a multifractal formalism. To achieve this, we compute explicitly the multifractal spectrum of such typical measures μ. This spectrum appears to be linear with slope 1, starting from 0 at exponent 0, ending at dimension d at exponent d, and it indeed coincides with the Legendre transform of the L^q-spectrum associated with typical measures μ.

Concerning the Hölder spectrum of continuous functions monotone in several variables, we find an upper bound valid for all functions of this type, and we prove that this upper bound is reached for generic functions monotone in several variables. For such generic functions $f:[0,1]^d \to \mathbb{R}$, we have $d_f(h) = d-1+h$ for all $h \in [0,1]$, and in addition, we obtain that the set E_f^h is empty as soon as $h > 1$. We also investigate the level set structure of such functions.

Diophantine approximation and Hausdorff dimension
Yann BUGEAUD

The theory of Hausdorff dimension and of Hausdorff measure is a crucial tool for proving the existence of real numbers with prescribed Diophantine properties. We explain this and survey various metrical results in Diophantine approximation.

Random fractals generated by uniform spacings
Claire COIFFARD MARRE

We study multivariate uniform spacings in $[0,1]^d$. Our interest is particularly devoted to "large" spacings. We consider the sets of exceptional points in the neighborhood of which such spacings are, infinitely often, unusually large. Our main result shows that these sets constitute random fractals, whose Hausdorff dimensions are explicitly evaluated.

Existence results for Plateau's problem in low dimensions?
Guy DAVID

The lecture should be centered on the question of existence of a minimal set of dimension 2 spanned by a given curve. I will try to convince the audience that the problem is unsolved and interesting, but very few results are known.

Trace results on tree-shaped domains with fractal self-contacting boundary
Thibault DEHEUVELS

In the present work, we focus on a class of tree-shaped ramified domains Ω with self-similar fractal boundary Γ^∞, that can be seen as a bidimensional idealization of the bronchial tree. This type of fractal tree was first studied by Mandelbrot and Frame. The work deals with trace theorems for functions in $W^{1,q}(\Omega)$, $1 < q < \infty$. Emphasis is put on the case when the domain is not an $\varepsilon - \delta$ domain as defined by Jones, and the fractal set is not totally disconnected. In this case, the classical trace results cannot apply.

The space of traces for functions in $W^{1,q}(\Omega)$ has been characterized in [1] using the Lipschitz spaces with jumps recently introduced by Jonsson. We investigate if the $B_s^{q,q}(\Gamma^\infty)$ regularity of a function can be characterized using the coefficients of its expansion in the Haar wavelet basis, *i.e.* if the space $B_s^{q,q}(\Gamma^\infty)$ coincides with the Lipschitz space with jumps $JLip(s,q,q;0;\Gamma^\infty)$.

When Γ^∞ is totally disconnected, this question has been positively answered by Jonsson for all s, q, $0 < s < 1$ and $1 \leq q < \infty$. Here, we distinguish two cases depending on the angle of the similitudes associated with Γ^∞, and we fully answer the question in the case when $0 < s < 1$ and Γ^∞ is connected.

[1] Y. ACHDOU, N. TCHOU, *Trace theorems for a class of ramified domains with self-similar fractal boundaries*, SIAM J. Math. Anal. 42, 2010.

SLE, KPZ and Liouville Quantum Gravity
Bertrand DUPLANTIER

(Joint work with Scott Sheffild, MIT)

Liouville quantum gravity is a way to produce a "random geometry" from the two-dimensional Gaussian free field (GFF). One replaces the Euclidean area measure dz on a smooth planar domain D with a (suitably regularized) random measure $\mu_\gamma = e^{\gamma h(z)} dz$, where $\gamma \in [0, 2)$ is a fixed constant and h is an instance of the zero or free boundary GFF on D.

The Knizhnik-Polyakov-Zamolodchikov (KPZ) relation then relates the Euclidean and quantum fractal dimensions of random subsets of D. This survey talk sketches a general and probabilistic proof of this relation.

We also present a connection between Liouville quantum gravity and Schramm-Loewner evolution (SLE): conformally welding two boundary arcs of a Liouville quantum gravity random surface generates SLE. A theory of quantum fractal measures (consistent with the KPZ relation) can then be developed and their evolution under welding analyzed via SLE martingales. As an application, quantum length and boundary intersection measures on the SLE curve itself can be constructed.

Multivariate Davenport series
Arnaud DURAND

Multivariate series of the form $\sum_n a_n \{n \cdot x\}$, where $x \in \mathbb{R}^d$ and $n \in \mathbb{Z}^d$, and $\{u\}$ is the sawtooth function, are the natural multidimensional extension of the standard Davenport series (which are a standard tool in analytic number theory, recently developed e.g. by R. de la Bretèche and G. Tenenbaum). We shall describe their pointwise regularity and their multifractal properties. This study calls upon new results that we shall also detail concerning the size and large intersection properties of the set of points in $\mathbb{R}^d$ that are approximated at certain rates by families of hyperplanes. A comparison will be drawn with Lévy random fields, which we also studied recently and present strong similarities with the above situation. This is a joint work with Stéphane Jaffard.

Estimating the intermittency coefficient of a random cascade
Laurent DUVERNET

A crucial parameter for multifractal random models is the so-called intermittency coefficient, which measures the curvature of the scaling exponent of the process: in the monofractal case, the scaling exponent is linear, and this coefficient is zero. The most commonly used method for recovering this coefficient from the data consists in approximating the theoretical moments of the process by empirical moments (structure functions). However, as was already apparent in some previous works (*e.g.* Ossiander and Waymire, 2000) this leads to a convergence rate for the statistical estimation procedure that is polynomially slower than the usual parametric rate $n^{1/2}$. Others estimators considered in the literature are based on the moments of the increments' (or wavelet coefficients') logarithms. Placing myself in the setting of Mandelbrot's b-adic cascades and Bacry, Delour and Muzy's log-normal MRW processes, I show that this leads indeed to better estimation procedures : the rates are $O((n \log^p n)^{1/2})$ for some positive integer p. Finally, I propose a new consistent estimator that attains the $n^{1/2}$ convergence rate.

Self-affine sets and measures
Kenneth FALCONER

I will survey some background and recent results on dimensions of self-affine sets and measures.

Multifractal analysis of Bernoulli convolutions associated with Salem numbers
DeJun FENG

For $\lambda \in (1, 2)$, the Bernoulli convolution ν_λ is the distribution of the random series $\sum_{n=1}^{\infty} \pm\lambda^{-n}$, where the signs $\pm$ are chosen independently with probability $\frac{1}{2}$. We show that when λ is a Salem number, ν_λ satisfies the multifractal formalism on $(0, \infty)$. Furthermore the range of local dimensions of ν_λ contains a non-trivial interval for a family of Salem numbers. We also show that absolutely continuous self-similar measures may have rich multifractal structures.

Fast change point analysis on the Hurst index of piecewise fractional Brownian
Mehdi FHIMA

(joint work with Pierre, R. BERTRAND and Arnaud GUILLIN)

In this presentation, we introduce a new method for change point analysis on the Hurst index for a piecewise fractional Brownian motion. We first set the model and the statistical problem. The proposed method is a transposition of the FDpV (Filtered Derivative with p-value) method introduced for the detection of change points on the mean in Bertrand et al. (2011) to the Hurst index. The underlying statistics of the FDpV technology is a new statistic estimator for Hurst index, so-called Increment Bernoulli Statistic (IBS). Both FDpV and IBS are methods with linear time and memory complexity, with respect to the size of the series.

Keywords: Change point analysis, Filtered derivative with p-value method, Hurst parameter, Increment Bernoulli Statistic, piecewise fractional Brownian motion.

References

[1] Bertrand, P. R., Fhima, M. and Guillin, A. (2011) Off-line detection of multiple change points by the Filtered Derivative with p-Value method, *To appear Sequential Analysis*.

Self-Excited Multifractal Model for Financial Return Fluctuations
Vladimir A. FILIMONOV

Scale invariance phenomena of many complex systems could not be characterized by only one intrinsic scale and requires the so-called multifractal description – introducing the continuous spectra of scales. Such multifractal properties are found in a wide range of objects in different domains of science. Started with the models of velocity increments and energy dissipation in developed turbulence, nowadays multifractal models are used to describe such critical systems as triggered seismicity in geophysics, asset return fluctuations in financial markets, traffic flow in computer networks and healthy human heart-beat rhythm in biology.

The first model that exhibits explicit time dependence was the so-called Multifractal Random Walk (MRW) that has been introduced by Bacry, Delour and Muzy. It was the only continuous stochastic stationary causal process with exact multifractal properties and Gaussian infinitesimal increments. However, although this model has no demerits inherent in the preceding models, it looses its sense for high-order incremental moments. The continuous-time Quasi-Multifractal model proposed by Saichev and Sornette and then extended by Saichev and Filimonov was free of this drawback. Being the development of the random walk model, it included several additional significant parameters, which make it possible to avoid the disadvantages of the MRW model.

The generalities of existing models described above are their "exogenous" form, which means that all of them are driven by external noise sources and do not describe the influence of past realization of the process onto to the future values – the mechanism which is essential for such applications as earthquakes and financial markets. In this work we would like to introduce the first "endogenous" multifractal model, which has explicit feedback of past values onto the future ones. The paper presents the new Self-Excited Multifractal (SEMF) model, discusses its novelty and key features and also describes the properties of the model.

With only three parameters the SEMF model generates the most important stylized facts of empirical financial returns, such as long-range dependence, leverage effect, multifractal scaling of moments and fat tails of the probability distribution. In addition it allows for efficient statistical estimation and provides a natural framework to distinguish between the impact of the exogenous news and the endogenous dynamics.

The dimension of a generic continuous function
Jonathan FRASER

I will discuss the problem of calculating the 'generic' dimension of a continuous function on [0,1]. I will focus on two approaches, namely, 'Baire category' and 'prevalence'. I will survey previous results and then present some new results which provide a complete answer to this question.

Large-Deviations Properties of Heart Rate: Estimation and Interpretation
Paulo GONCALVES

(joint work with P. Loiseau, C. Médigue, N. Attia, S. Seuret, F. Cottin, D. Chemla, M. Sorine and J. Barral)

Abstract : In the realm of multiscale analyses applied to heart beat rate variability, multifractal analysis provides with a natural and rich framework to measure the roughness of a time series. As such, it drawn special attention of both mathematicians and practitioners, and led them to characterize relevant physiological factors impacting the heart rate variability. Notwithstanding these considerable progresses, multifractal analysis almost exclusively developed around the concept of Legendre singularity spectrum – for which efficient and elaborate estimators exist – but which are structurally blind to subtle features, like non-concavity or, to a certain extent, non scaling of the distributions. Large deviations theory allows for bypassing these limitations but it is only very recently that performing estimators were proposed to reliably compute the corresponding large deviations singularity spectrum. In this talk, we illustrate the relevance of this approach, on both theoretical objects and on human heart rate signals stemming from real study cases. As expected, we will show that large deviations principles reveal significant information that otherwise remains hidden with classical approaches, and which can be reminiscent of some physiological characteristics.

Riemann-Liouville Multifractional Stable Process : simulation via the Haar basis.

Julien HAMONIER

The *Riemann-Liouville Multifractional Stable Process* (RLMSP in short) is a Symmetric α-Stable ($\mathcal{S}\alpha\mathcal{S}$) process ($1<\alpha<2$), denoted by $\{R(t)\}_{t\in[0,1]}$ and defined for all $t\in[0,1]$ as the stochastic integral :

$$R(t) := \int_0^1 (t-s)_+^{H(t)-1/\alpha} Z_\alpha(ds), \tag{1}$$

where $x_+ = \max\{x,0\}$, $H(\cdot)$ is a continuous function on $[0,1]$ with values in a compact set included in $(1/\alpha,1)$ and $Z_\alpha(\cdot)$ is a $\mathcal{S}\alpha\mathcal{S}$ random measure on $\mathbb{R}$ with Lebesgue measure as its control measure.

By expanding for every fixed $t\in[0,1]$, the kernel function $s\mapsto (t-s)_+^{H(t)-1/\alpha}$ on the Haar basis of $L^2[0,1]$, we can show that the process $\{R(t)\}_{t\in[0,1]}$ can be represented as the following random series :

$$R(t) = \frac{t^{1+H(t)-1/\alpha}}{1+H(t)-1/\alpha}\epsilon_{-1} + \sum_{j=0}^{+\infty}\sum_{k=0}^{2^j-1} \epsilon_{j,k}\theta_{j,k}^{\alpha}(t), \tag{2}$$

where $\{\epsilon_{-1}\}\cup\{\epsilon_{j,k} : j\in\mathbb{Z}_+, 0\le k\le 2^j-1\}$ denotes a sequence of identical distributed $\mathcal{S}\alpha\mathcal{S}$ random variables and

$$\theta_{j,k}^{\alpha}(t) = \frac{2^{j(1-1/\alpha)}}{1+H(t)-1/\alpha}\left\{\left(t-\frac{2k+2}{2^{j+1}}\right)_+^{1+H(t)-1/\alpha} - 2\left(t-\frac{2k+1}{2^{j+1}}\right)_+^{1+H(t)-1/\alpha} + \left(t-\frac{2k}{2^{j+1}}\right)_+^{1+H(t)-1/\alpha}\right\}.$$

Then we make a precise study of the convergence of the series in (2). To this end, we use some technics inspired by [LA] as well as a nice result of [RS93] which allows to bound the tail of the distribution of the supremum of the absolute value of the stable process. Finally, thanks to representation (2) of RLMSP, we introduce a new way to simulate this process.

[LA] Linde, W., Ayache, A., *Series Representations of Fractional Gaussian Processes by Trigonometric and Haar Systems*, Electronic Journal of Probability, 14, 94, 2691-2719, 2009.

[RS93] Rosinski, J., Samorodnitsky, G., *Distributions of Subadditive Functionals of Sample Paths of Infinitely Divisible Processes* The Annals of Probability, 21, 2,996-1014, 1993.

Local limit theorem of some additives function of local time of multifractional Brownien motion
Ouahhabi HANAE

In this talk we consider $X^H = (X^{H(t)}(t), t \in \mathbb{R}^+)$ a multifractional Brownian motion (mBm) with Hurst functional $H(.) \in \mathcal{C}^\beta(R+, (0,1))$. We will define some additives functions of local time of X^H, we study they Hˆlderien regularity in time, and mixed regularity, that's will alow us to give some property of moduli of continuity. In the end we give local limit theorem of this additive function; but before that we have to prove they verify the local asymptotic self-similarity. This work was inspired from the one of Boufoussi et *al.* [1] .

[1] B. Boufoussi, M. Dozzi, and R. Guerbaz (2007), *Sample path properties of the local time of multifractional Brownian motion,* Bernoulli 133,pp. 849-867

Multifractal analysis of the divergence of Fourier series
Yanick HEURTEAUX

A famous theorem of Carleson says that, given any function $f \in L^p(\mathbb{T})$, $p \in (1, +\infty)$, its Fourier series $(S_n f(x))$ converges for almost every $x \in \mathbb{T}$. Beside this property, the series may diverge at some point, without exceeding $O(n^{1/p})$. We define the divergence index at x as the infimum of the positive real numbers β such that $S_n f(x) = O(n^\beta)$ and we are interested in the size of the exceptional sets E_β, namely the sets of $x \in \mathbb{T}$ with divergence index equal to β. We show that quasi-all functions in $L^p(\mathbb{T})$ have a multifractal behavior with respect to this definition. Precisely, for quasi-all functions in $L^p(\mathbb{T})$, for all $\beta \in [0, 1/p]$, E_β has Hausdorff dimension equal to $1 - \beta p$. We also investigate the same problem in $\mathcal{C}(\mathbb{T})$, replacing polynomial divergence by logarithmic divergence. In this context, the results that we get on the size of the exceptional sets are rather surprizing. This is a joint work with Frédéric Bayart (Clermont-Ferrand).

Remarks on limit sets of infinite IFSs
Martial HILLE

For a (finite) iterated function system there is a unique compact invariant set which is also the attractor of the IFS. For an infinite IFS the attractor is in general not closed, and hence there are two different sets associated to an infinite IFS, namely the (not necessarily closed) attractor and its closure.

Since the influential contributions of Mauldin&Urbanski, the attractor of an infinite IFS is referred to as limit set, and, for example, the connection of the pressure function and the Hausdorff dimension of the limit set has been established.

In this talk we present new results on the difference between the attractor and its closure, both in terms of sets and in terms of Hausdorff dimensions.

Differential 1-forms on fractals and harmonic spaces
Michael HINZ

The talk is concerned with substitutes for 1-forms on spaces on which no classical derivation exists. We briefly explain two approaches.
The first origins in simple ideas from combinatorial geometry and is available on sufficiently simple fractals that allow an approximation by regular cell complexes.
The second works whenever a harmonic structure is given, for instance on finitely ramified fractals or on abstract harmonic spaces. It uses a specific tensor structure and connects analysis (Hodge decomposition) and topology (Čech cohomology) in a very nice way.

Besicovitch-Federer projection theorem and geodesic flows on Riemann manifolds
Risto HOVILA

The Besicovitch-Federer pro jection theorem states that a set $E \subset \mathbb{R}^n$ E with $\mathcal{H}^m(E) < \infty$ is purely m-unrectifiable, if and only if its projection on almost every m-plane V in the Grassmannian manifold $G(n,m)$ has zero $\mathcal{H}^m$ measure. This theorem was first proved by Besicovitch in [1] for purely 1-unrectifiable planar sets and generalized to arbitrary n and m by Federer in [2]. We extend this theorem to the case where the orthogonal projections are replaced by a transversal family of continuously di?erentiable mappings $\{P_\lambda : \mathbb{R}^n \to \mathbb{R}^m\}_{\lambda \in \Lambda}$. The definition of transversality was introduced by Peres and Schlag in [3]. As an application of this result we show that on certain class of Riemann surfaces with constant negative curvature and with boundary there exist natural 2-dimensional measures invariant under the geodesic flow having 2-dimensional supports such that their pro jections to the base manifold are 2-dimensional but their supports are Lebesgue negligible.

This is joint work with Esa Järvenpää, Maarit Järvenpää and François Ledrappier.

References :

[R1] A. S. Besicovitch, On the fundamental geometrical properties of linearly measurable plane sets of points (III) - Math. Ann. 116 (1939), 349–357.

[R2] H. Federer, The (φ, k) rectifiable subsets of n space - Trans. Amer. Math. Soc. 62 (1947), 114–192.

[R3] Y. Peres and W. Schlag, Smoothness of projections, Bernoul li convolutions, and the dimension of exceptions - Duke Math. J. 102 (2000), 193–251

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p-spectrum of Random Wavelet Series

Stéphane JAFFARD

Multifractal analysis is concerned with the determination of the dimensions of the sets of points where the regularity exponent of a function (or a measure) takes a given value. For locally bounded functions, the natural notion of "regularity exponent" is supplied by the Hölder exponent. However, in most applications in signal and image processing, the local boundedness hypothesis is not met (as shown recently by P. Abry, S. Jaffard, S. Roux and H. Wendt) and extensions of the Hölder exponent have to be used. A natural one is supplied by the "p-exponent", initially introduced by A. Calderon and A. Zygmund in the 50s. Though a multifractal formalism adapted to this setting had been proposed by C. Melot and S. Jaffard, no natural classes of random models presenting the following features were known up to now: -their sample paths are a.s. non-locally bounded, -they are multifractal, in the sense that their p-spectrums are a.s. "non-trivial" (i.e. not supported by a single point). The purpose of this talk is to show that some extensions of Random Wavelet Series (a model initially introduced by J.-M. Aubry and S. Jaffard) present these features. The one-variable setting (random processes) and the multi-variate setting (random fields) will both be considered.

Projections of measures invariant under the geodesic flow

Maarit JARVENPAA

Based on a joint work with R. Hovila, E.Jarvenpaa and F.Ledrappier

Let M be a compact Riemann surface, let μ be a probability measure on the unit tangent bundle SM and let $\Pi : SM \to M$ be the natural projection. We denote by $\dim_H$ the Hausdorff dimension. Assuming that μ is invariant under the geodesic flow, F. Ledrappier and E. Lindenstrauss proved the following result for the projected measure $\Pi_*\mu$: if $\dim_H \mu \leq 2$, then $\dim_H \Pi_*\mu = \dim_H \mu$, and if $\dim_H \mu > 2$, then $\Pi_*\mu$ is absolutely continuous with respect to the 2-dimensional Lebesgue measure.

We will show that the absolute continuity fails at the threshold 2. More precisely, for any compact Riemann surface with negative curvature there exists a measure which is invariant and ergodic under the geodesic flow such that $\dim_H \Pi_*\mu = 2$ and $\Pi_*\mu$ is singular with respect to the 2-dimensional Lebesgue measure.

The graph and range singularity spectra of independent cascade functions

Xiong JIN

With the "iso-Hölder" sets of a function we naturally associate subsets of the graph and the range of the function. We compute the Hausdorff dimension of these subsets for a class of statistically self-similar multifractal functions, namely the b-adic independent cascade functions.

Local homogeneity and dimensions of measures in doubling metric spaces. Part II

Antti KAENMAKI

We introduce two new concepts, local homogeneity and local Lq - spectrum, both of which are tools that can be used in studying the local structure of measures. The main emphasis is given to the study of local dimensions of measures in doubling metric spaces. As an application, we reach a new level of generality and obtain new estimates for conical densities, in multifractal analysis, and on the dimension of porous measures. This is a joint work with V. Suomala and T. Rajala"

How large dimension guarantees a given angle?

Tamas KELETI

(joint work with *Viktor Harangi, Tamás Keleti, Gergely Kiss, Péter Maga, András Máthé, Pertti Mattila* and *Balázs Strenner*)

We study the following two problems:

(1) Given $n \geq 2$ and α, how large Hausdorff dimension can a compact set $A \subset \mathbb{R}^n$ have if A does not contain three points that form an angle α?

(2) Given α and δ, how large Hausdorff dimension can a compact subset A of a Euclidean space have if A does not contain three points that form an angle in the δ-neighborhood of α?

Some angles $(0, 60°, 90°, 120°, 180°)$ turn out to behave differently than other $\alpha \in [0, 180°]$.

Dimension gaps for Bernoulli approximations of conformal IFS

Marc KESSEBOHMER

Kifer, Peres and Weiss showed that the dimension of measures that make the digits of the continued fraction expansion i.i.d. variables are bounded above by $1 - 10^{-7}$. We discuss an approach to this problem which only relies on the multifractal formalism and an exhaustion principle. (Joint work with Bernd O. Stratmann and Mariusz Urbanski)

Packing dimension, packing dimension profiles, and Lévy processes

Davar KHOSHNEVISAN

We provide a Fourier-analytic formulation of Taylor's 1986 formula for the range of a Lévy process. Our formulation and methods rely on the properties of the Cauchy transform. We show, through examples, some applications of our formula. This part of the talk is based on joint work with Yimin Xiao.

We also extend the concept of packing dimension profiles, due to Falconer and Howroyd (1997) and Howroyd (2001), and use our extension in order to determine the packing dimension of an arbitrary image of a general Lévy process. This portion is based on collaboration with René Schilling and Yimin Xiao.

Quasisymmetric modification of metrics on self-similar sets
Jun KIGAMI

Using the notions of scales and their gauge functions associated with self-similar sets, we give a necessary and sufficient condition for two metrics on a self-similar set being quasisymmetric to each other. As an application, we construct metrics on the Sierpinski carpet which is quasisymmetric with respect to the Euclidean metrics and obtain an upper estimate of the conformal dimension of the Sierpinski carpet. Furthermore, we present a relation between the conformal and the sepctral dimensions of the Sierpinski carpet.

On the decomposition of the balls into finitely many congruent pieces
Gergely KISS

(joint work with M. Laczkovich)

A set called m-divisible if it can be decomposed into m pairwise disjoint congruent pieces. Van der Waerden noticed in 1949 that disc is not 2-divisible. S. Wagon proved is 1983 that the d-dimensional ball is not m-divisible for $m \leq d$. Wagon's result motivated the question whether or not the d-dimensional balls are m-divisible for $m > d$. In this paper we prove that if d is divisible by 3 then the d-dimensional balls are m-divisible if m is large enough. This implies that for every d there are d-dimensional convex compact sets which are m-divisible for some m. This answers a question of C. Richter.

Dimensions of deterministic and random code-tree fractals
Henna KOIVUSALO

The dimension of a self-affine fractal generated by one iterated function system (IFS) has long been known. In this work we study dimensional properties of code-tree fractals, a type of self-affine fractals generated by several affine IFSs.

Firstly, we calculate the dimension of an arbitrary fixed code-tree fractal. Then we define a new type of random fractal, which is not necessarily built in an i.i.d. manner, but still has a probabilistically self-repeating structure. This allows us to use suitable ergodic theorems in finding the almost sure dimension of the random code-tree fractal. In the dimension calculation, standard potential theoretic methods seem to apply.

This is a collaborate work with E. Jarvenpa, M. Jarvenpa, A. Kaenmaki, O. Stenflo and V. Suomala. At the time of the submission of this abstract the result concerning dimension of random code-tree fractals is still a work in progress.

Minkowski content and fractal curvature measures for self-conformal sets
Sabrina KOMBRINK

We investigate Minkowski measurability of invariant sets of conformal iterated function systems in $\mathbb{R}$. We show that every nonlattice invariant set is Minkowski measurable, whereas in the lattice situation both is possible – the Minkowski content might or might not exist. Furthermore, a complete characterisation on the existence of Winter's fractal curvature measures, which refine the notion of Minkowski content, is given.

Multi-operator scaling stable random fields
Céline LACAUX

The fractional Brownian motion [Kolmo,MVN] is a very powerful model in applied mathematics. However, it is isotropic and then it is not convenient for anisotropic media modeling. In order to describe the anisotropy of the media, [OSSRF] has introduced some anisotropic Gaussian and α-stable random fields, which satisfy the following operator scaling property

$$\forall c > 0,\ \left(X\left(c^E x\right)\right)_{x\in\mathbb{R}^d} \overset{(f.d.d.)}{=} c\left(X(x)\right)_{x\in\mathbb{R}^d} \tag{3}$$

with E a real-valued matrix of size $d \times d$ and $c^E = \exp\left(\log(c)E\right)$. Their Hölder regularity properties, which are characterized by the matrix E, may depend on the direction but do not vary along the trajectories. Then, to allow more flexibility, we introduce the local asymptotic operator scaling property which generalizes both the local asymptotic self-similarity property [BEJARO] and the operator scaling property (3). We then consider a class of Gaussian and stable random fields defined by an harmonizable representation, which includes the harmonizable fields studied in [OSSRF]. These fields are locally operator scaling of order $E(x)$. Moreover, their Hölder regularity properties and their anisotropic behavior at point x are characterized by the matrix $E(x)$.

This is a joint work [BLS11] with Hermine Biermé (MAP 5, Université Paris Descartes) and H.P. Scheffler (Fachbereich Mathematik, Universit at Siegen)).

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Fractal Strings and a Spectral Reformulation of the Riemann Hypothesis
Michel LAPIDUS

In [1] (J. Lond. Math. Soc., 1995), a spectral reformulation of the Riemann hypothesis was obtained by M. Lapidus and H. Maier, involving inverse spectral problems for fractal strings.

In short, one can always hear whether a given fractal string of dimension D (different from1/2) is Minkowski measurable if and only if the Riemann hypothesis is true. Later on, this work was revisited in light of the theory of complex dimensions of fractal strings developed by M. L. Lapidus and M. van Frankenhuijsen in [2] (Fractal Geometry and Number Theory, Birkhauser, 2000) and [3] (Fractal Geometry, Complex Dimensions and Zeta Functions, Springer, 2006; 2nd enl. ed. to appear in 2011). Moreover, in [3], the "spectral operator" was introduced as the operator that sends the geometry of a fractal string onto its spectrum. In the present work, joint with H. Herichi, we provide a rigorous functional analytic framework for the study of the spectral operator a. We show that a is an unbounded normal operator acting on a scale of Hilbert spaces (indexed by the Minkowski dimension D in (0,1) of the underlying fractal string) and precisely determine its spectrum (which turns out to the be equal to the range of values of the Riemann zeta function along the line Re s = D). Furthermore, we deduce that for a given D¿0, the spectral operator is invertible if and only if there are no Riemann zeros on the vertical line Re s = D. It follows that the associated inverse spectral problem has a positive answer for all possible dimensions D in (0,1), other than in the mid-fractal case when D = ?, if and only if the Riemann hypothesis is true. We also show rigorously that (as was suggested in [3]) the spectral operator has an operator-valued Euler product that is convergent (in a suitable sense, namely, in the strong operator topology) even inside the critical strip 0 ¡ Re s ¡1.

Self-similar sets and Martin boundaries

Ka-Sing LAU

The limiting behavior of a transient Markov chain can be described by its Martin boundary together with an elegant discrete potential theory. In this talk we consider different classes of Markov chains on the symbolic space of a self-similar set K. Our objective is to identify the Martin boundary with K, so as to induce an harmonic structure on K. This relates to the current interest on the analysis on fractals. The hyperbolic graphs and the hyperbolic boundaries also arises naturally in this study.

Self-similar structure on intersection of homogeneous symmetric Cantor sets

Wenxia LI

For a homogeneous symmetric Cantor set C, we consider all real numbers t such that the intersection $C \cap (C + t)$ is a self-similar set and investigate the form of the corresponding iterated function systems.

Dimension of level sets of multiple ergodic averages - an example

Lingmin LIAO

We propose to study multiple ergodic averages from multifractal analysis point of view. As an attempt, we are interested in a special example. Consider the symbolic space $\mathbb{D} = \{+1, -1\}^{\mathbb{N}}$ and the following level set

$$B_\theta := \{x \in \mathbb{D} : \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} x_k x_{2k} \cdots x_{\ell k} = \theta\}.$$

By using Riesz products, we prove that the Hausdorff dimension of B_θ is given by the formula

$$1-\frac{1}{\ell}+\frac{1}{\ell}H\Big(\frac{1+\theta}{2}\Big), \quad \text{with } H(t)=-t\log_2 t-(1-t)\log_2(1-t).$$

This is a joint work with A.-H. Fan and J.-H. Ma.

An Algorithm to compute the centered Hausdorff measure of self-similar sets
Marta LLORENTE

(joint work with Manuel Morán)

We provide an algorithm to compute the centered Hausdorff mesure, C^s, of self-similar sets satisfying the strong separation condition. We prove the convergence of the algorithm and test its efficciency with some examples.

Intersection of self-conformal sets of equal dimension
Andras MATHE

We study the intersection of two self-conformal sets of equal Hausdorff dimension in a Euclidean space. Under various assumptions we prove that the intersection has positive measure if and only if its relative interior (with respect to one of the self-conformal sets) is non-empty. We also provide necessary algebraic conditions for the generators when the intersection is of positive measure.

(Joint work with M. Elekes and T. Keleti.)

Singular integrals on fractal subsets of Heisenberg groups
Pertti MATTILA

Based on a joint work with Vasilis Chousionis I shall discuss results saying that several singular integral operators are unbounded on many fractal type subsets of Heisenberg groups. Similar results in Euclidean spaces have been known before. I shall also discuss some applications to removable singularities of the sub-Laplacian equation.

Mixing effective and classical notions of dimensions.
Dan MAULDIN

We discuss some of the effective notions of measure and dimension. We will illustrate how one can use these notions and some techniques involving them in proving the following result. For $0<\alpha<1$, let E_α consist of all number x in the interval [0,1] with constructive dimension α. Let C be the classical Cantor middle third set. THEOREM. Let $1-dim_H C\le\alpha\le 1$. Then for any Martin-Lof random real $r\in[0,1]$, $dim_H[(C+r)\cap E_\alpha]=\alpha-1+dim_H(C)$. The method of proof involves Kolmogorov complexity, effective versions of one of Marstand's results and effective versions of some results from additive number theory.

Multifractal analysis of a fractal boundary: the example of the Knopp function
Clothilde MELOT

(Joint work with Mourad Ben Slimane)

Fractal boundaries are often used in modelization of complex phenomena for example in turbulence, diffusion problems and quantum physics. A usual classification tool to study a fractal interface is the computation of its fractal dimension. But a new method developped in [JaffHeur] propose to compute pointwise exponents which describe the behavior of the interface locally.

In this communication we propose to apply it to the graph of the Knopp function, i.e we will study the graph of

$$F : [0,1] \to \mathbb{R}$$
$$x \mapsto \sum_{j\geq 0} \sum_{k=0}^{2^j-1} 2^{-\alpha j} \Lambda(2^j x - k)$$

with $0 < \alpha < 1$ and $\Lambda(x) = \inf(x, 1-x)$.

Whereas the Knopp function as a function of x is a very regular function since it has everywhere the same regularity, we will show that the regularity of the graph, seen as the boundary of a 2D domain, changes from point to point. Using the characterization of the maxima and minima done in [DD], we will show how the pointwise exponents can be computed.

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Statistically subsets of self-affine fractals
Jun-Jie MIAO

We studied a class of fractals which are random subsets of self-affine fractals. We will give an expression for the 'almost sure' Hausdorff dimension of these sets.

A Spectral Approach to Survival Probabilities in Porous Media
Thanh Binh NGUYEN

We consider a diffusive process in a bounded domain with heterogeneously distributed traps, reactive regions or relaxing sinks. This is a mathematical model for chemical re- actors with heterogeneous spatial distributions of catalytic germs, for biological cells with specfi c arrangements of organelles, and for mineral porous media with relaxing agents in NMR experiments. We propose a spectral approach for computing survival probabilities which are represented in the form of a spectral decomposition over the Laplace operator eigenfunctions. We illustrate the performances of the approach by considering diffusion in- side the unit disk

filled with reactive regions of various shapes and reactivities. Finally, we show that signi

cant improvements in computational time and accuracy can be achieved for rotation invariant reactive regions.

This is a joint work with D. Grebenkov.

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Multifractal tube formulas: multifractal zeta-functions, explicit formulas and weak convergence of tube measures.

Lars OLSEN

Tube formulas give the volume of the set of all points within the distance r of a (compact) subset of Euclidean space for $r \geq r$, and play an instrumental role in the study of geometric measure theory. Recently, very precise tube formulas for self-similar sets satisfying the Open Set Condition (OSC) have obtained by, for example, Gatzouras, Lapidus and Winter.

We study multifractal tube formulas of self-similar sets and self-similar measures satisfying the OSC.

Multifractional Stochastic Volatility Models: Simulation via the Haar Basis

Qidi PENG

First we define the Multifractional Stochastic Volatility Models (MSVM), denoted by Z(t) and explain the notations behind them. Then using the Haar basis of L2[0,1], we introduce a random series representation of Z(t). Moreover we show that this series is with probability 1, convergent in all the Banach spaces of Holder functions over [0,1] of order r¡1/2 and give an estimation of the rate of convergence. Finally, by making use of the latter representation of Z(t) we simulate this process.

A multifractal analysis for which b and B differ

Jacques PEYRIERE

An example of measure on [0,1] for which Hausdorff and packing dimensions of the level sets of its local Hölder exponents are given by the Legendre transforms of the b and B functions (which differ, except at q=0 or q=1).

Escape rates for Gibbs measures and conformal repellers
Mark POLLICOTT

Let T:X -¿ X be a conformal repeller (e.g., a hyperbolic Julia set) and let m be the associated Gibbs measure. For any point x in X we can consider the dimension of the subset X(r) of points which never enter the r-ball around x. The ratio of the difference of the dimensions of X and X(r) divided by the measure of the ball converges to a constant, which we identify.

This is joint work with Andrew Ferguson.

Random affine IFS
Michal RAMS

I'll present a result on the dimension of the limit set for a class of random self-affine iterated function systems.

Lognormal scale invariant random measures
Rémi RHODES

We consider the continuous analog of the celebrated Mandelbrot star equation with lognormal weights. We show existence and uniqueness of measures satisfying the aforementioned continuous equation; these measures fall under the scope of the Gaussian multplicative chaos theory developped by J.P. Kahane in 1985 (or possibly extensions of this theory). As a by product, we also obtain an explicit characterization of the covariance structure of these measures. This is a joint work with R. Allez and V. Vargas.

Curvature-direction mesaures of self-similar sets
Tilman ROTHE

(joint work with Martina Zähle)

We obtain fractal curvature-direction measures for a large class of self-similar sets F in $\mathbb{R}^d$. Such measures jointly describe the distribution of normal vectors and localize curvature by analogues of the higher order mean curvatures of differentiable submanifolds. They decouple as independent products of the unit Hausdorff measure on F and a self-similar measure on the sphere, which can be computed by an explicit formula and admits an interpretation as density. This local approach uses an ergodic dynamical system formed by extending the code space shift by a subgroup of the orthogonal group.

Burkholder functionals, Morrey's question and singular integrals
Eero SAKSMAN

We consider norms of singular integral operators, quasiconvexity of variational integrals, a question of Morrey, and sharp regularity estimates of quasiconformal maps. All these different themes will be linked via Burkholder functionals. The talk is based on joint work with K. Astala (Helsinki), S. Geiss (Innsbruck) , T. Iwaniec (Syracuse), S. Montgomery-Smith (Missouri) and I. Prause (Helsinki).

Probabilistic and Dynamical Covering–A Survey
Joerg SCHMELING

(Joint work with Ai-Hua Fan)

The talk will be a survey of classical and recent results on a class of questions that originated in probability and then found its counterpart in ergodic theory. We will give motivations of these questions as well as illustrate the main methods in this theory. A variety of applications, especially in Diophantine approximation, will be discussed.

Let X be a set and $T : X \to X$ a map. The investigation of the dynamics (X, T) consists in studying the behavior of the iterates $T^n x$ $(n = 0, 1, 2, \cdots)$, called orbit of x, for different initial points x. If X is equipped with a σ-field $\mathcal{B}$ and a T-invariant measure defined on $\mathcal{B}$, Poincaré recurrence theorem states that for any measurable set $A \in \mathcal{B}$ the following recurrence holds:

$$T^n x \in A \quad \text{i.o.} \quad \mu\text{-a.e. } x \in A$$

where "i.o." stands for "infinitely often" meaning for infinitely many n. Instead of A, we consider a sequence of measurable sets $\{A_n\}$. A natural problem is to find conditions which ensure the following generalized recurrence:

$$T^n x \in A_n \quad \text{i.o.} \quad \mu\text{-a.e. } x \in X.$$

For a given point x which represents an orbit, we define

$$\mathcal{T}(x, \{r_n\}) := \{y \in X : T^n x \in B(y, r_n) \ \text{ i.o.}\}$$

It is the set of the targets y hit by x. For a given target y, we define

$$\mathcal{O}(y, \{r_n\}) := \{x \in X : T^n x \in B(y, r_n) \ \text{ i.o.}\}$$

It is the set of the orbits x hitting the target y. The roles of x and y are not symmetric. These are two different problems to study $\mathcal{T}(x, \{r_n\})$ and $\mathcal{O}(y, \{r_n\})$. The first will be named the covering problem and the second the shrinking target problem.

For a fixed sequence (r_n), we will write

$$\mathcal{T}(x, \{r_n\}) = \{y \in X : x_n \in B_{r_n}(y) \ \text{ i.o.}\}. \ \text{ and } \ \mathcal{C}(y, \{r_n\}) = \{x \in X^{\mathbb{N}} : x_n \in B_{r_n}(y) \ \text{ i.o.}\}.$$

Question 1. When $X = \mathcal{T}(x, \{r_n\})$?

Question 2. How about the covering numbers $\sum_{k=1}^n 1_{C(y_r, r_k}(x_k)$?

Question 3. Suppose X is compact. What is the first n such that $X = \bigcup_{k=1}^n C$

Most of cases examined in this paper are special cases of the following setting. We take $\{x_n\}$ to be a stationary process $(\xi_n)_{n\geq 0}$ defined on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$, taking values in a metric space (X, d).

Coupon collector's problem The well known coupon collector's problem corresponds to $X = \{1, 2, \cdots, m\}$ is a finite set and $\mathbb{P}$ is the Bernoulli product measure on $\{1, 2, \cdots, m\}^{\mathbb{N}}$. We will discuss a more general Markov covering.

Dvoretzky random covering problem The Dvoretzky random covering problem corresponds to $X = \mathbb{T}$ is the circle and $\{\xi_n\}$ is an independent and identically and uniformly distributed random variables.

Dynamical covering problem By dynamical covering we refer to the case where $(X, \mathcal{B}, \mu, T, d)$ is measure-theoretic dynamical system and the stationary process (ξ_n) is defined by the orbit $\xi_n(x) = T^n x$.

Geometry of the common dynamics of Pisot substitutions with the same incidence matrix.

Tarek SELLAMI

Any Pisot substitution can be associated with a bounded set with interesting properties, called the Rauzy fractal. This set is obtained by projection of the broken line associated with an infinite fixed point. Two substitutions having the same incidence matrix can have different Rauzy fractals. We show that under weak conditions, the intersection of these two fractals has strictly positive measure, and can also be generated by a substitution.

Fractals which are not tube-null and projections of random measures.

Pablo SHMERKIN

A set $E \subset \mathbb{R}^d$ is called **tube-null** if for every $\varepsilon > 0$ there are lines $\{L_i\}_{i=1}^{\infty}$ and numbers $\{w_i\}_{i=1}^{\infty}$ such that $\sum_i w_i < \delta$ and E is covered by the union of the tubes of width w_i around L_i. All tube-null sets are null, but not vice-versa.

Tube-null sets arise naturally in the study of the localization properties of the Fourier transform. Motivated by this, A. Carbery and coauthors asked several questions on the structure of the sets which are *not* tube-null. Perhaps the most basic of these is: how small can the Hausdorff dimension of a non-tube-null set be?

By adapting a method of Y. Peres and M. Rams to study projections of certain random measures, we are able to answer all these questions. In particular, we show there are non-tube-null sets of dimension $d-1$ (it is easy to see any set of dimension smaller than $d-1$ is tube-null). Along the way, we study geometric properties of a class of random measures which exhibits strong self-similarity.

This is joint work with Ville Suomala.

Algebraic difference of random Cantor sets

Károly SIMON

The study of the algebraic difference

$$F_2 - F_1 = \{y - x : x \in F_1, y \in F_2\}$$

of two dynamically defined Cantor sets $F_1, F_2 \subset \mathbb{R}$ was motivated by the research of Palis and Takens. Palis conjectured that if

$$\dim_{\mathrm{H}} F_1 + \dim_{\mathrm{H}} F_2 > 1$$

then *generically* it should be true that

$$F_2 - F_1 \text{ contains an interval.}$$

In this talk we consider the same problem for some families of random Cantor sets. The talk is based on a recent joint work with M. Dekking and B. Székely.

Fractals invariant under the multiplicative integers
Boris SOLOMYAK

Motivated by questions on the multiftactal analysis of multiple ergodic averages, studied by A.-H. Fan, L. Liao, and J. Ma, we investigate the dimension properties of a class of sets invariant under the action of the semigroup of multiplicative integers, given by $T_k : (x_n) \mapsto (x_{kn})$, where k is a positive integer. A representative example is the set of all 0-1 sequences (x_n) such that x_n, x_{2n} are not both equal to 1. We compute the Hausdorff and Minkowski dimensions of these sets and show that they are typically different.

This is joint work with R. Kenyon and Y. Peres.

Fractals arising in percolation
Jeffrey STEIF

2-dimensional critical percolation on the hexagonal lattice and its dynamical counterpart lead to a number of interesting fractal sets, which are either subsets of $\mathbb{R}^2$ or of $\mathbb{R}$. These various fractal sets will be discussed.

Algebraic growth rates for zonal Kleinian groups
Bernd STRATMANN

The talk explains how infinite ergodic theory can be employed to derive estimates for the algebraic growth rate of the Poincaré series of a Kleinian group with parabolic elements at its critical exponent of convergence.

Local homogeneity and dimensions of measures in doubling metric spaces. Part I

Ville SUOMALA

We introduce two new concepts, local homogeneity and local Lq - spectrum, both of which are tools that can be used in studying the local structure of measures. The main emphasis is given to the study of local dimensions of measures in doubling metric spaces. As an application, we reach a new level of generality and obtain new estimates for conical densities, in multifractal analysis, and on the dimension of porous measures. This is a joint work with A. Käenmäki and T. Rajala"

Quantitative recurrence properties in beta dynamical system and continued fraction system

Eva THEUMANN-WESFREID [a,b]

(joint work withVéronique BILLAT [c])

The aim of this study was to detect randomness in heart rate (HR) and respiratory frequency (Rf) during a high altitude mountain ascent, by investigating the changes in the fractal scaling and time-frequency behavior and the entropy of HR and Rf with altitude. In order to analyze simultaneously these two signals, we first used the local cosine4 orthonormal bases whose spectrum is not redundant as those computed with the short Fourier transform. The signal energy is therefore split into local cosine4 spectrum energies over time segmented boxes.We performed then, the detrended fluctuations and the wavelet leader scaling analysis (DFA, WLA) to estimate HR and Rf scaling exponents. Results showed that HR and Rf were differently affected by altitude. Indeed, the average HR was not modified by altitude (p=0.18) in contrast to Rf which increased significantly (p = 0.0003). The variability of HR was altered during the mountain ascent after 3000m of altitude, the short range HR DFA (1.0 - 1.5) exponents increased after the altitude of 3000m (p¡0.03). In contrast, the Rf DFA short range exponents ($0.5-1$), were not affected by altitude (p=0.67). The ratio of low frequencies (0.94%) cosine4 spectrum energy over the local spectrum energy of HR did not change with altitude (p=0.11) in contrast with the Rf which decreased with altitude from 71 to 65

Keywords: Fractal analysis; Detrended Fluctuation Analysis (DFA); Wavelet; Heart rate, Entropy, Hypoxia, Fatigue.

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Transfinite Hausdorff dimension

Mariusz URBANSKI

Making extensive use of small transfinite topological dimension trind, we ascribe to every metric space X an ordinal number (or -1 or Ω) tHD(X), and we call it the transfinite Hausdorff dimension of X. This ordinal number shares many common features with Hausdorff dimension. It is monotone with respect to subspaces, it is invariant under bi-Lipschitz maps (but in general not under homeomorphisms), in fact like Hausdorff dimension, it does not increase under Lipschitz maps, and it also satisfies the intermediate dimension property. The primary goal of transfinite Hausdorff dimension is to classify metric spaces with infinite Hausdorff dimension. As our main theorem, we show that for every countable ordinal number α there exists a compact metric space X_α (a subspace of the Hilbert space l_2) with $\mathrm{tHD}(X_\alpha) = \alpha$ and which is a topological Cantor set, thus of topological dimension 0. In our proof we construct metric versions of Smirnov topological spaces and establish several properties of transfinite Hausdorff dimension, including its relations with classical Hausdorff dimension.

Characterization of self-similar and anisotropic images with 2D hyperbolic wavelets
Béatrice VEDEL

This is a joint work (still in progress) with M. Clausel, S. Roux, P. Abry and S. Jaffard.

Anisotropic images -that is images having different geometric characteristics along different directions - naturally appear in various areas (biomedical, hydrology, geostatistics and spatial statistics...) ([1], [2], [3], [4]...). The detection and characterization of the anisotropy is an important issue. ([2],[5]...). In our work, we consider a specific model of anisotropic textures defined in [3], which are of particular interest because of their self-similarity properties. Using an Hyperbolic wavelet Transform, we are able to characterize an anisotropy and self-similarity of these textures.

Simulations have been performed to confirm the theoretical results. They provide estimators of an anisotropy matrix and the Hurst index of the model. The estimation procedures have numerically shown to be efficient even for images of small size. This study is planned to be generalize to other anisotropic models.

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Quantitative recurrence properties in beta dynamical system and continued fraction system
Baowei WANG

Let $(X, T, \mathcal{B}, \mu)$ be a measure theoretic dynamical systems. Consider the metrical properties of following recurrence set and shrinking target problem, called also as dynamical diophantine approximation:

$$\left\{x \in X : |T^n x - x| < \varphi(n, x), \text{ for infinitely many } \mathbb{N}\right\}$$

$$\left\{x \in X : |T^n x - x_0| < \varphi(n, x), \text{ for infinitely many } \mathbb{N}\right\}$$

where φ is some positive function.

In this talk, we will discuss the size of above sets in the setting of two irregular systems: beta dynamical system and continued fraction system. The main idea used here is to approximate the system by sub-systems with regular properties.

Meanwhile, we would also like to pose the following question: In the classic diophantine approximation, the mass transference principle developed by V. Beresnevich and S. Velani is a powerful tool to determine the size of limsup sets. Is it possible to develop similar principles for the size of above diophantine type sets driven by a dynamical system?

Bi-Lipschirz equivalence of self-similar sets.

Zhi-Ying WEN

Doubling Measures and Nonquasisymetric Maps on Whitney Modification Sets in Euclidean Spaces

Zhi-Xiong WEN

Gaussian Random Fields: Spectral Measures and Fine Properties

Yimin XIAO

Let $X = \{X(t), t \in \mathbb{R}^N\}$ be a centered Gaussian random field with stationary increments. Its spectral measure Δ can either be absolutely continuous (a familiar example is that of fractional Brownian motion) or singular with respect to the Lebesgue measure. In this talk we present some recent results on regularity and fractal properties of X in terms of the asymptotic properties of Δ.

The Lipschitz equivalence between fractals

Xiong YING

Local and global curvatures for classes of fractals

Martine ZAHLE

An approach to local and global fractal Lipschitz-Killing curvatures of self-similar (random) sets and certain other types of fractals via ergodic theorems for associated dynamical systems is presented. The latter provide the tool for deriving average limits for the rescaled curvatures of small parallel sets. For non-arithmetic logarithmic contraction ratios of the generating function systems we obtain ordinary limits of the global variants using asymptotic uniform distribution properties of related probability laws.

Equilibrium states - Rigidity of dimension

Anna ZDUNIK

(joint results with M. Szostakiewicz and M. Urbanski)

Bieberbach coefficients for SLE$_\kappa$.

Michel ZINSMEISTER

We revisit Bieberbach conjecture about coefficients of injective holomorphic mappings in the disk within the framework of SLE processes of O. Schramm.

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