Large Deviations Properties of Heart Rate Variability

Paulo Gonçalves

DMASC
(N. Attia, J. Barral, D. Chemla, F. Cottin, P. Loiseau, C. Médigues, S. Seuret, M. Sorine)

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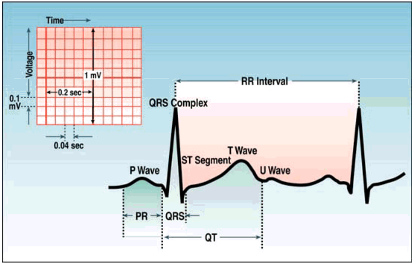
Outline

RR signals

ECG



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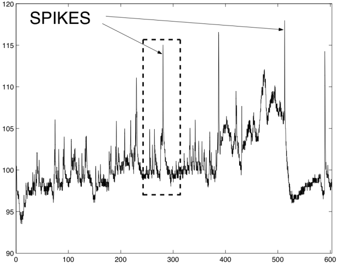

Rich structures :

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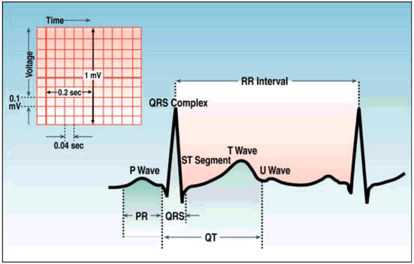

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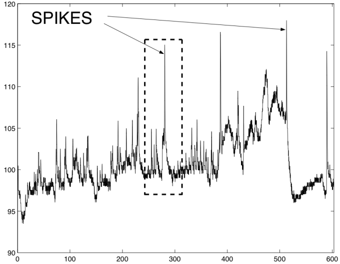

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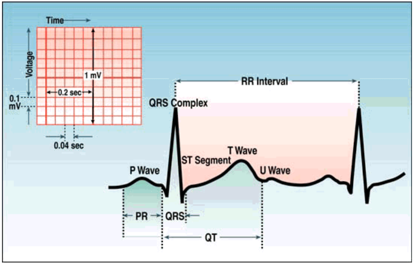

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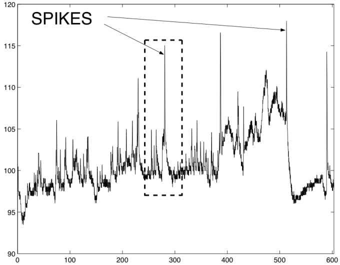

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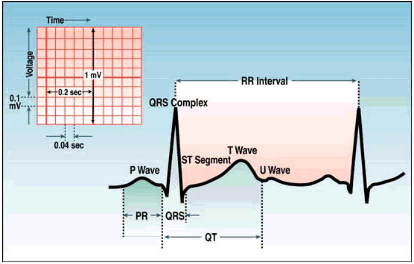

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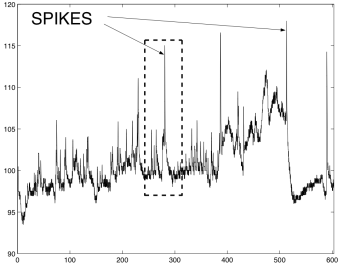

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- scale invariance properties. . .

Scale Invariance & Related work

Huge amount of results based on scale invariance properties of RR signals

- long range dependency (1/f *analysis of HRV*, Kobayashi, 1982)
- chaotic dynamical systems & strange attractors ('89 and controversial topic in *Chaos*, 2009)
- fractal dimensions of trajectories (entropy based measurement of system complexity, 90's)
- local Hölder regularity estimation (Pincus, 94)
- multiractal analysis of RR time series (P. Ivanov et al. very active in the period 1995–2005)

Applied to a wide range of physiological studies

- aging and gender effect characterization
- sleep apnea detection
- infarction recovery
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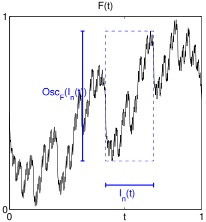
Multifractal spectra

Notations and Definitions :



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$$O_F(I) = \sup_{s\in I} F(s) - \inf_{s\in I} F(s)$$

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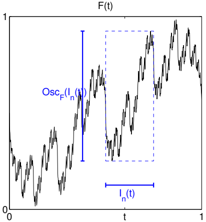

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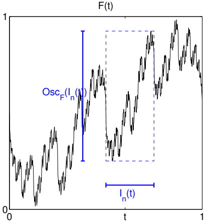

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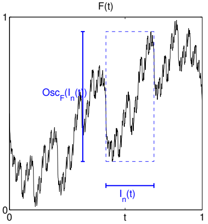

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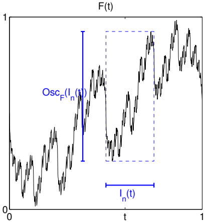

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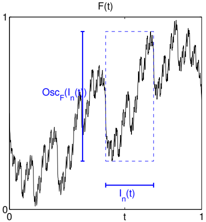

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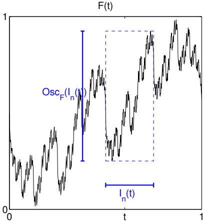

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Large Deviations Spectrum : Estimation

From Large Deviations Theory, for each q where τ_O is differentiable,

$$f_O(\alpha(q)) = \tau^*(\alpha(q)) \text{ where } \alpha(q) = \tau_O'(q)$$

Find a positive sequence $(\varepsilon_n(q))_{n\geq 1}$ converging to 0 and such that :

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Theorem ("On the estimation of the Large Deviations spectrum", J. Barral, P. G., *J. stat. Phys.*, 2011)

Let $q \in \mathbb{R}$ and $\eta > 0$. Suppose $\rho_n = |\tau_{O,n}(q) - \tau_O(q)| + \sup_{q' \in [q-\eta, q+\eta]} |\tau_{O,n}'(q') - \tau_O'(q')|$ converges to 0 and $\tau''(q)$ exists. Let $(\epsilon_n(q))_{n\geq 1}$ be a positive sequence converging to 0 such that $\max(1/\sqrt{n}, \sqrt{\rho_n}) = o(\epsilon_n(q))$ as $n \to \infty$. Then

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A sufficient condition :

$$\lim_{n\to\infty} \frac{n\,\epsilon_n(q)^2}{-2\tau_{O,n}''(q)} = \infty$$

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A good choice : $\epsilon_n(q) = \sqrt{-2\tau''_{O,n}(q)/n\log(2)}$

Consistent from an **Estimation** viewpoint, since the measure :

$$\mu_{q,n}(I) := \mathbf{1}_{O(I)>0} O(I)^q \cdot 2^{n\tau_{O,n}(q)} \Rightarrow \begin{cases} \mathbb{E}_{\mu_{q,n}}(\alpha(I)) = \tau'_{O,n}(q) \\ \epsilon_n(q) = \sqrt{-2\mathbb{V}\mathrm{ar}_{\mu_{q,n}}(\alpha(I))} \end{cases}$$

Possible refinements :

- Under additional conditions on the convergence speed of $\tau_{O,n} \to \tau_O$:

$$\epsilon_n(q) = \sqrt{-2(1+\lambda)\tau''_{O,n}(q)\log(n)/n\log(2)}$$

- When the sequence $(\log O(I_n(t)) - \log O(I_{n-1}(t)))$ is close to stationary w.r.t μ_q and if μ_q is sufficiently mixing :

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Possible refinements :

- Under additional conditions on the convergence speed of $\tau_{O,n} \to \tau_O$:

$$\epsilon_n(q) = \sqrt{-2(1+\lambda)\tau''_{O,n}(q)\log(n)/n\log(2)}$$

- When the sequence $(\log O(I_n(t)) - \log O(I_{n-1}(t)))$ is close to stationary w.r.t μ_q and if μ_q is sufficiently mixing :

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Large Deviations Spectrum : Estimation

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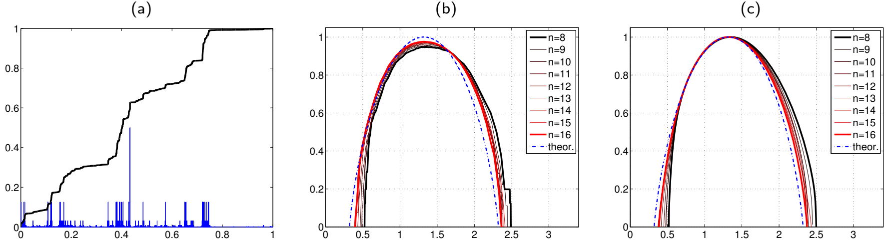
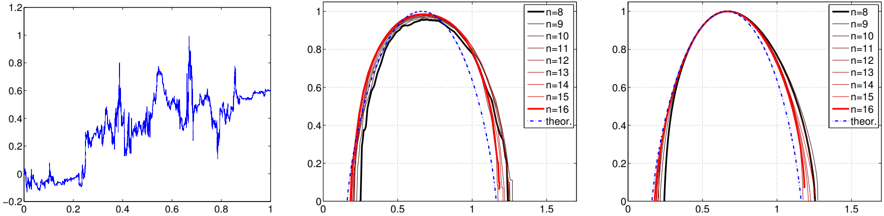
Multifractal objects

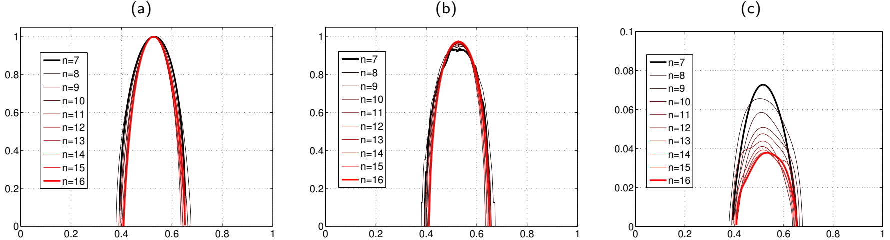


(a) Binomial multiplicative cascade. (b) Large deviations, $\epsilon_n(q) = \sqrt{-2\tau''_{\mathrm{Osc},n}(q)\log\log(n)/n\log(2)}$. (c) Legendre.

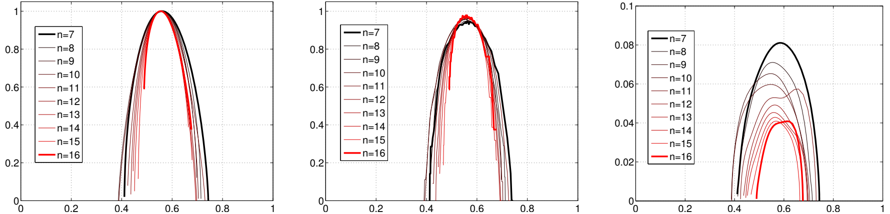


(a) Brownian motion in multifractal time (b) Large deviations, $\epsilon_n(q) = \sqrt{-2\tau''_{\mathrm{Osc},n}(q)\log\log(n)/n\log(2)}$. (c) Legendre

Monofractal objects

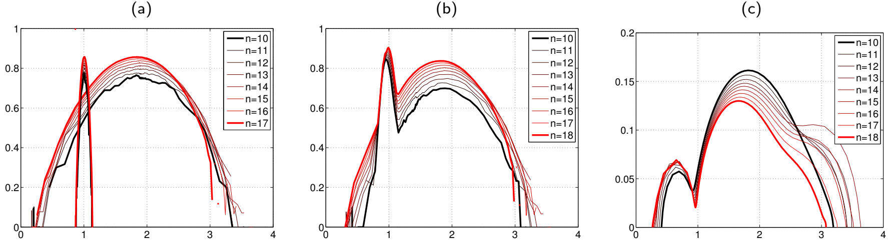


Standard Brownian Motion of Hurst exponent. (a) Legendre spectrum estimates. (b) LDS estimates, $\epsilon_n(q) = \sqrt{-2\tau''_{\mathrm{Osc},n}(q)/n\log(2)}$; (c) evolution of $\epsilon_n(q)$ with $\alpha_n(q)$.



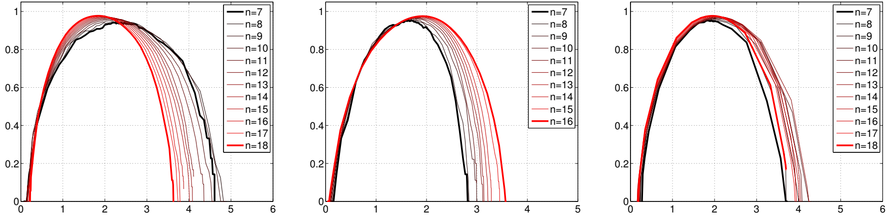
Dyadic synthesis of a multiplicative signed cascade with $H = 0.55$. (a) Legendre. (b) LDS estimates, $\epsilon_n(q) = \sqrt{-2\tau''_{\mathrm{Osc},n}(q)/n\log(2)}$; (c) evolution of $\epsilon_n(q)$ with $\alpha_n(q)$.

Non concave and non scale invariant spectra



Concatenation of two lognormal multiplicative cascades.

(a) Individual LDS estimates. (b) LDS estimated from the concatenated trace. (c) Evolution of $\epsilon_n(q) = \sqrt{-2\tau''_{\mathrm{Osc},n}(q)/n\log(2)}$



Non scaling measures. Multiplicative cascades with non identically distributed random vectors.

$$\epsilon_n(q) = \sqrt{-2\tau''_{\mathrm{Osc},n}(q)/n\log(2)}.$$

Data Base

To illustrate that :

- Large Deviations Spectra convey important information on the analyzed signals
- The proposed estimation algorithm is adaptive and efficient

We used RR signals from an heterogeneous database, characteristic of different HRV profiles :

- Control subjects
- Diabetic subjects
- Heart failure subjects (I or II in the NYHA)

All nightly records, of length $N = 2^{13}$ data points (observation scale $J = 13$)

relative scale	nb. of RR in dyad. int.	nb dyad. int.	physiol. oscillation	
$J-1$	3	4096		
$J-2$	5	2048	respiration	
$J-3$	9	1024		baroreflex
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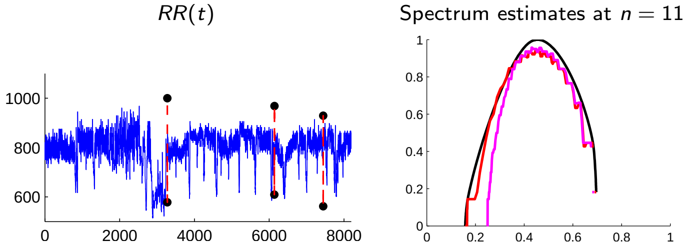
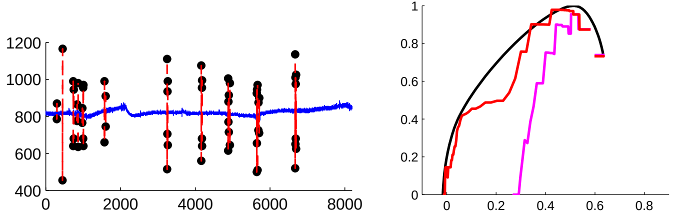
Non concave spectrum : Extrasystoles

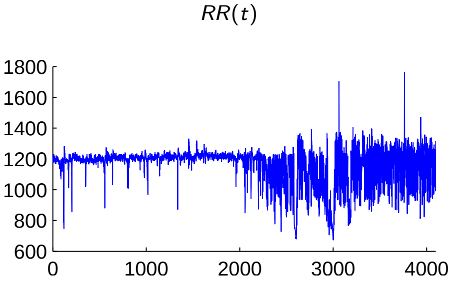


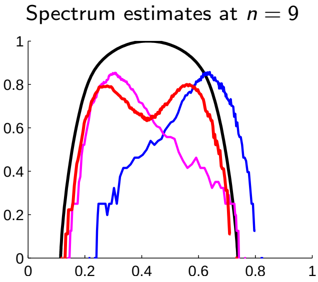
Diabetic subject. LDS estimates with extrasystoles and without extrasystoles. Legendre spectrum estimate.



Heart failure subject. LDS estimates with extrasystoles and without extrasystoles. Legendre spectrum estimate.

Non concave spectrum : Phase change

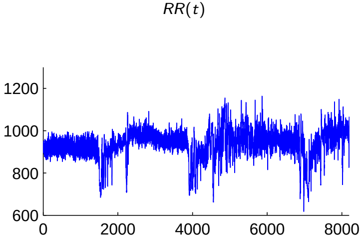


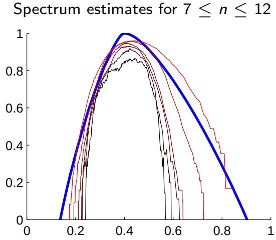


Control subject. Record overlapping two different sleep regimes. LDS estimates over the full period ; over the first regime only ($t < 2000$) ; over the second regime only ($t > 2000$).

Legendre spectrum estimate.

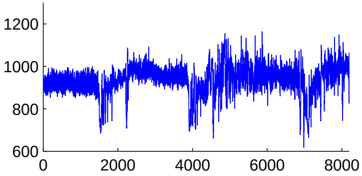
Scaling versus non scaling ?



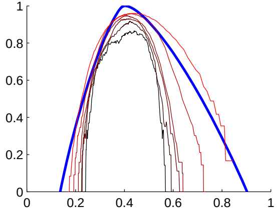


Scaling versus non scaling ?

$RR(t)$



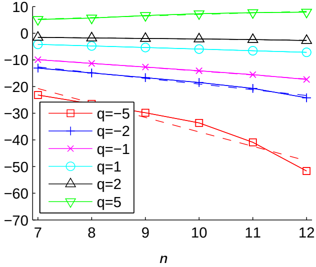
Spectrum estimates for $7 \leq n \leq 12$



This non superimposition of the spectra is also reminiscent in the non linearity of the functions $\tau_{O,n}(q)$ (for $q < 0$)

.

$n\tau_{O,n}(q)$



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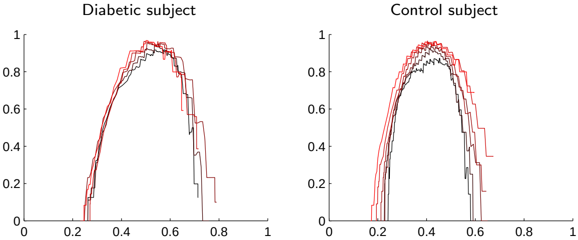

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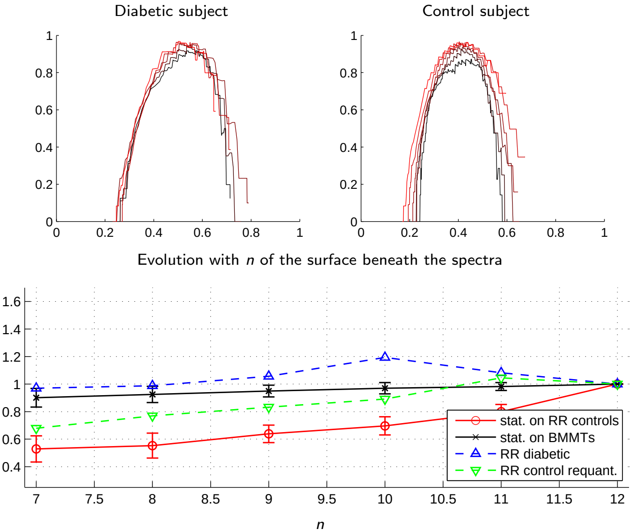

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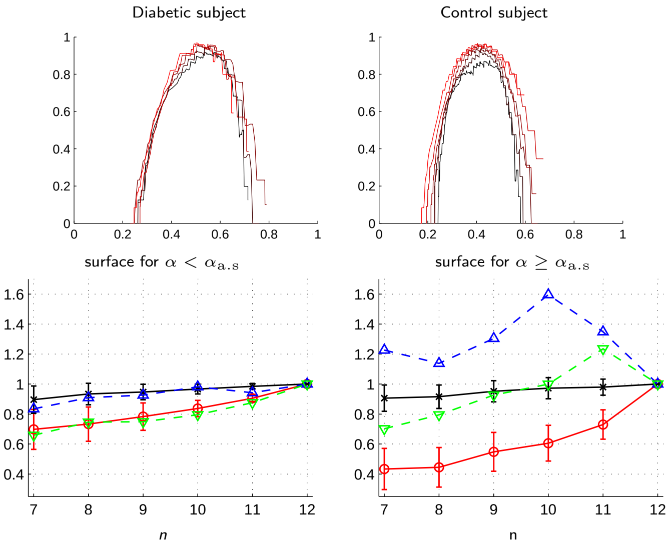

Concluding remarks and perspectives

- Methodology
  - First Large Deviations Spectrum estimate that is mathematically sound
  - Adaptive (non-parametric) estimator of multifractal spectrum
  - A complementary tool to more popular approaches (Legendre spectrum)
  - Renew the focus on scaling characteristics
  - Exhibits richer singularity structures
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  - Non concave spectra are the signature of different physiological factors : extrasystole and their density, (*intermittency*), phase changes,. . .
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